Information Design in Collective Decision Games

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  - designs information structure to manipulate beliefs
- Question: **What is the optimal information structure?**

Whom to persuade and How?

- Optimal Information Structure depends on:
  1. Set of information structures available to the designer:
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  2. The Voting rule
- Compare expected payoffs in equilibrium to analyze:
  - Which player will the designer target?
  - Will she include the most difficult to convince?
  - Which voting rule is least vulnerable to influence?

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- A voting rule is given. (# of votes required)
- Designer chooses an information structure
- State of the world is realized
- Players observe the signal; update beliefs
- They play a BNE of the induced game.

The Committee

- Each member wants to match state & alternative
- *Differ* in cost of mismatch:

$$u_i(x,\theta) = \begin{cases} -q_i & \text{if} \quad x = x_1, \theta = \theta_0 \\ -(1-q_i) & \text{if} \quad x = x_0, \theta = \theta_1 \\ 0 & \text{otherwise} \end{cases}$$

- $q_i \in (0,1)$ is called the "threshold of doubt".
- Higher $q_i \Rightarrow$ more difficult to convince
- **i votes for x_1 if belief on $\theta_1 > q_i$**

The Information Designer

- Always wants the outcome to be x_1
- Designs *information structure* $\{T, \pi\}$
  1. Finite Realization Space: T
  2. Conditional Distribution Functions: $\pi : \Theta \to \Delta(T)$
- Only restriction on signals – *Bayes' Consistency*
- Solves the problem:

$$\begin{array}{rl} \max & \Pr(\mathit{outcome} = x_1) \\ \text{Subject To} & \textit{Incentive Constraints} \end{array}$$

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- **Question**: *Can the designer do better?*

Can the Designer Do Better?

Table : Designer's Expected Payoff

Information Structure	Majority Rule	Unanimity
No Information	0	0
Full Information	0.3	0.3
Public Signal		
Private Independent		
Correlated		

Public Signal

- Same signal = same posterior
- But behavior *differs* because of *different* q_i
- Reduces to one player problem
- $q_1 < q_2 < q_3 \Rightarrow$ Designer makes the marginal player indifferent.

Optimal Information Structure

Proposition

Given a voting rule, the optimal information structure of the designer, with a public signal, is characterized by $\{T, \pi\}$ *with* $T = \{t_0, t_1\}$ *and* $\pi : \Theta \to \Delta(T)$ *is defined as:*

$$\pi(t_0|\theta_0) = p_k \quad \text{and} \quad \pi(t_0|\theta_1) = 0$$

$$\pi(t_1|\theta_0) = 1 - p_k \quad \text{and} \quad \pi(t_1|\theta_1) = 1$$

$$1 - p_k = \frac{\psi_1}{\psi_0}\left(\frac{1 - q_k}{q_k}\right)$$

where q_k *is the threshold of doubt of k-th voter, and k is the number votes required for* x_1 *be chosen as the outcome.*

All in a Day's Work

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Private Independent		
Correlated		

Private Information

- Signals are private, conditionally independent, identically distributed.
- No longer a one player problem!
- Two effects:
  1. Signals diverge – bad for designer
  2. Strategic Voting – (potentially) good for designer
- Being *pivotal* carries additional information.
- "Potentially" good – can infer a bad signal
- Is there some way to make strategic voting *good*?

Go for the Easiest!

- To kill the bad signals: *convert all rules to unanimity*
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Designer as Mediator

- Designer can send arbitrarily correlated signals
- Think of the designer as making "recommendations"
- Optimizing over: distributions of *action* profiles

$$\sigma : \Theta \to \Delta(\mathcal{A})$$

- Solution concept – Bayes Correlated Equilibrium

Main Results

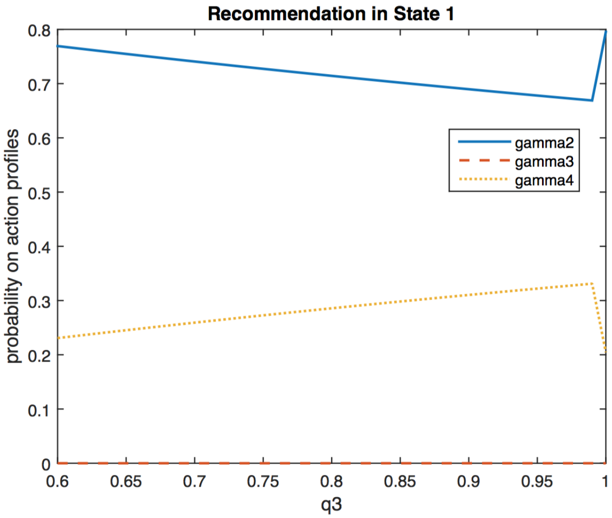
Theorem (1)

Under non-unanimous voting rules, using a public signal is sub-optimal for the designer.

- There exist information structure with private correlated signals that give the designer a higher expected payoff.
- Designer does not target the marginal player!
- Calls upon the more-difficult-to-convince in the good state.

Light, Shade and Perspective...

Figure : Illustrative Example



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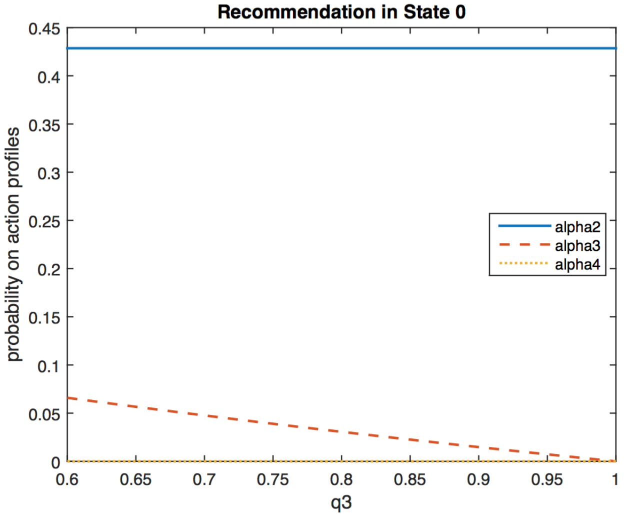

Main Results

Theorem (2)

Under the unanimity rule, the optimal information structure of the designer is such that:

$$\sigma(x_1, ..., x_1|\theta_1) = 1 \quad \text{and} \quad \sigma(x_1, ..., x_1|\theta_0) = \left(\frac{\psi_1}{\psi_0}\right)\left(\frac{1-q_n}{q_n}\right)$$

And the designer's expected utility is $\frac{\psi_1}{q_n}$.

United We Stand.

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Public Signal	0.6	0.5
Private Signal	0.6	0.5
Correlated Signals	0.6462	0.5

That's all Folks!

- Two main results:
  1. Public Information is suboptimal for designer under non-unanimous voting rule
  2. Unanimity is least vulnerable to influence.
- Two main contributions:
  1. Bayesian Persuasion with Strategic interaction
  2. Private and Correlated Signals
- Two closest papers:
  1. Wang (2015)
  2. Alonso-Camara (2015)